A New Perspective on Uncalibrated Photometric Stereo

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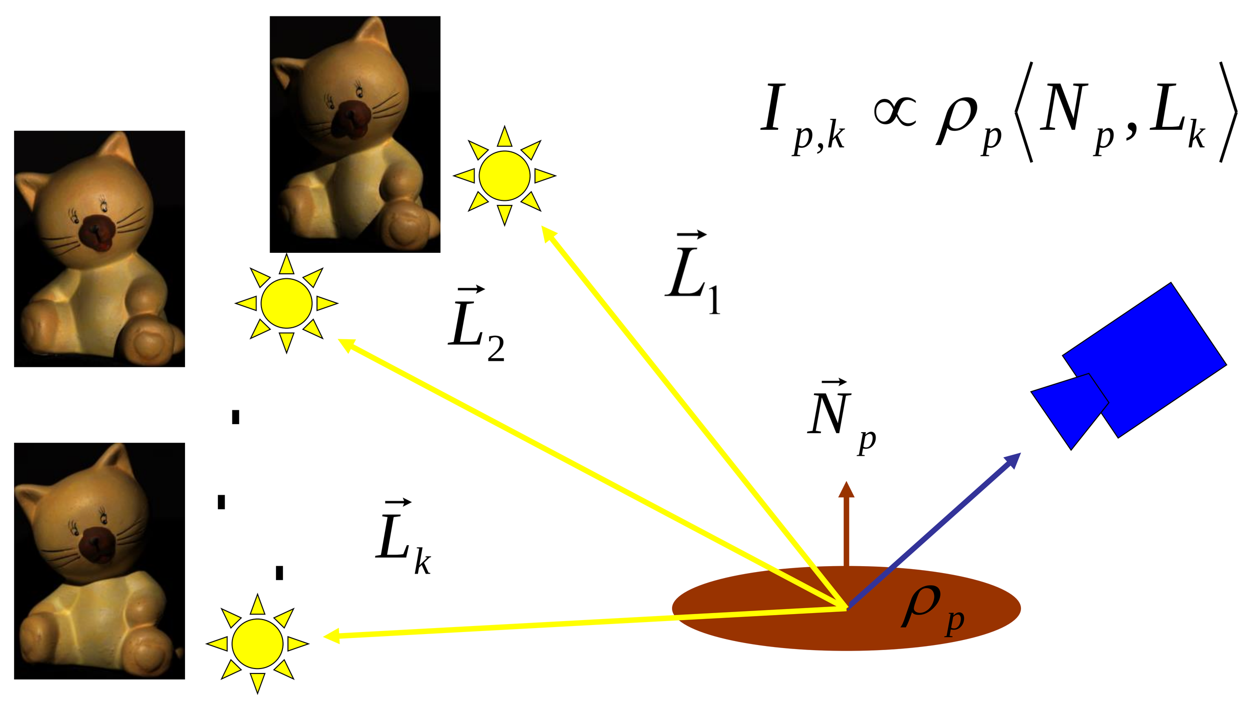
Uncalibrated Photometric Stereo



Our contribution
We show that under the perspective projection model, uncalibrated photometric stereo has a unique solution under the integrability constraint.

Uncalibrated Photometric Stereo (UPS)
Determine normals ($\vec{N}$), albedo map (r) and lights ($\vec{L}_1 \cdots \vec{L}_k$) given images ($I_1 \cdots I_k$).

Orthographic Uncalibrated Photometric Stereo

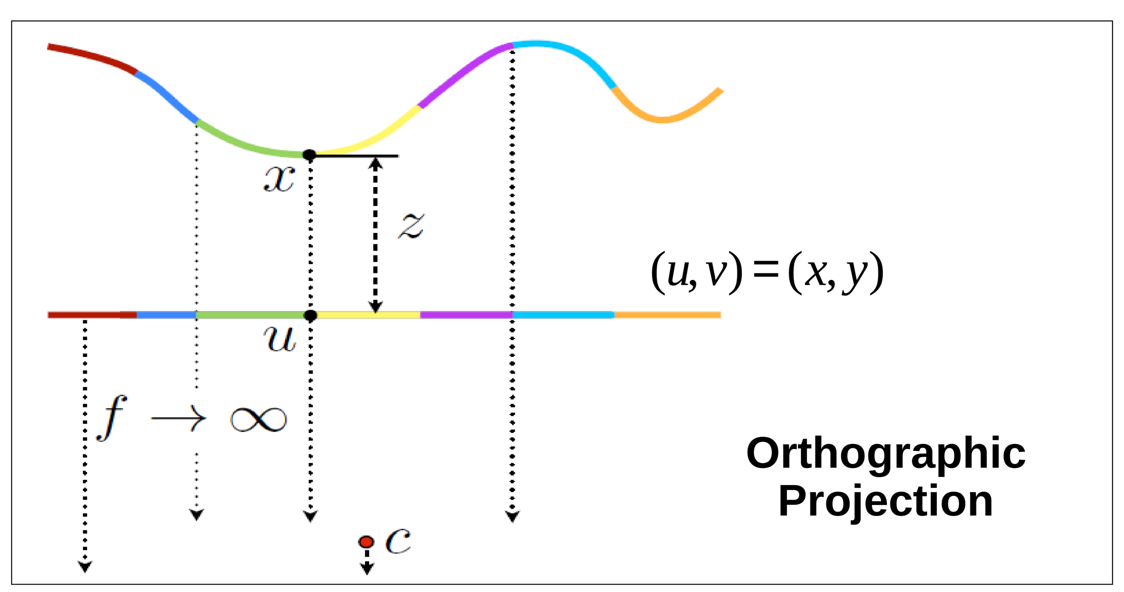
Reconstruction is possible (via the integrability constraint of the normal map) up to the Generalized Bas Relief (GBR) ambiguity [Belhumeur, Kriegman & Yuille 1997].

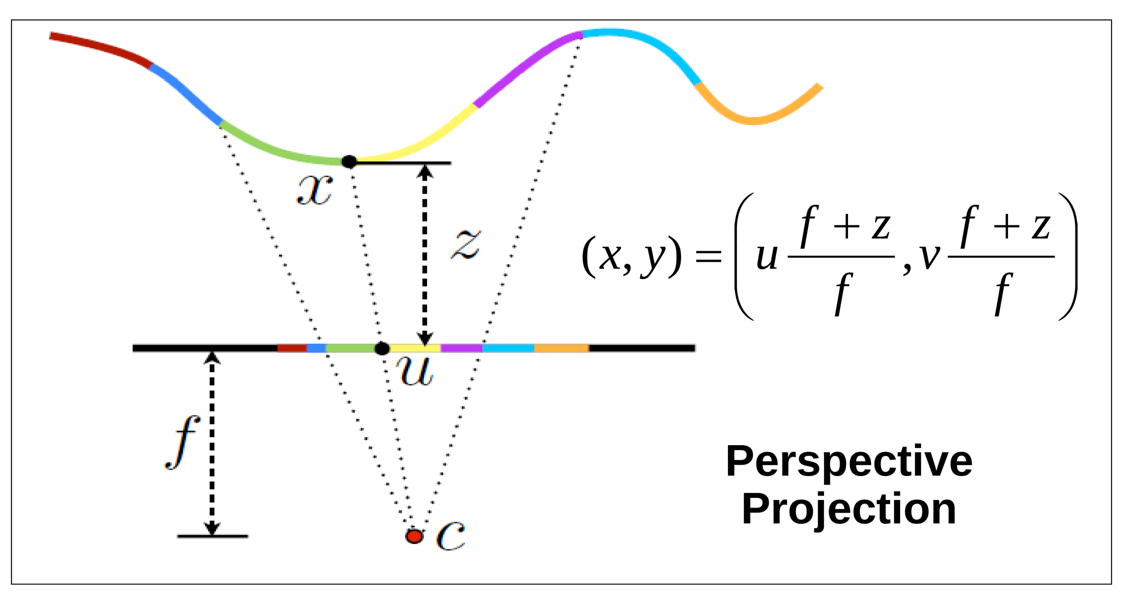
$$I_{p,k} = B_p^T S_k = r_p N_p^T Q^{-1} Q L_k \qquad Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & \nu & \lambda \end{bmatrix}$$

To be determined via additional constraints

Limitation of prior works: to solve the remaining ambiguity, stringent assumptions are imposed on the objects or how the images are captured

Orthographic versus Perspective Projection Model





Same image formation model by reparametrizing the normal map.

$$I_k = r\frac{p p_k + q q_k + 1}{\|L_k\|\sqrt{p^2+q^2+1}} e_k \quad \text{where} \quad N \propto \begin{bmatrix} p \\ q \\ -1 \end{bmatrix} \text{and } L_k \propto \begin{bmatrix} p_k \\ q_k \\ -1 \end{bmatrix}$$

Perspective Uncalibrated Photometric Stereo

Theorem: The integrability constraint in the case of perspective projection is sufficient to uniquely identify the normals. The reconstructed normals can be computed via:

N x 9 Perspective integrability matrix:

$$P \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} = 0 \qquad \text{where} \qquad P \doteq \begin{bmatrix} B_u^T \hat{B} & B_v^T \hat{B} & \frac{u}{f} B_u^T \hat{B} - \frac{v}{f} B_v^T \hat{B} \end{bmatrix}$$

Skew symmetric matrix

Same 6 columns as in the orthographic case [Belhumeur, Kriegman & Yuille, The Bas-Relief Ambiguity, 1997].

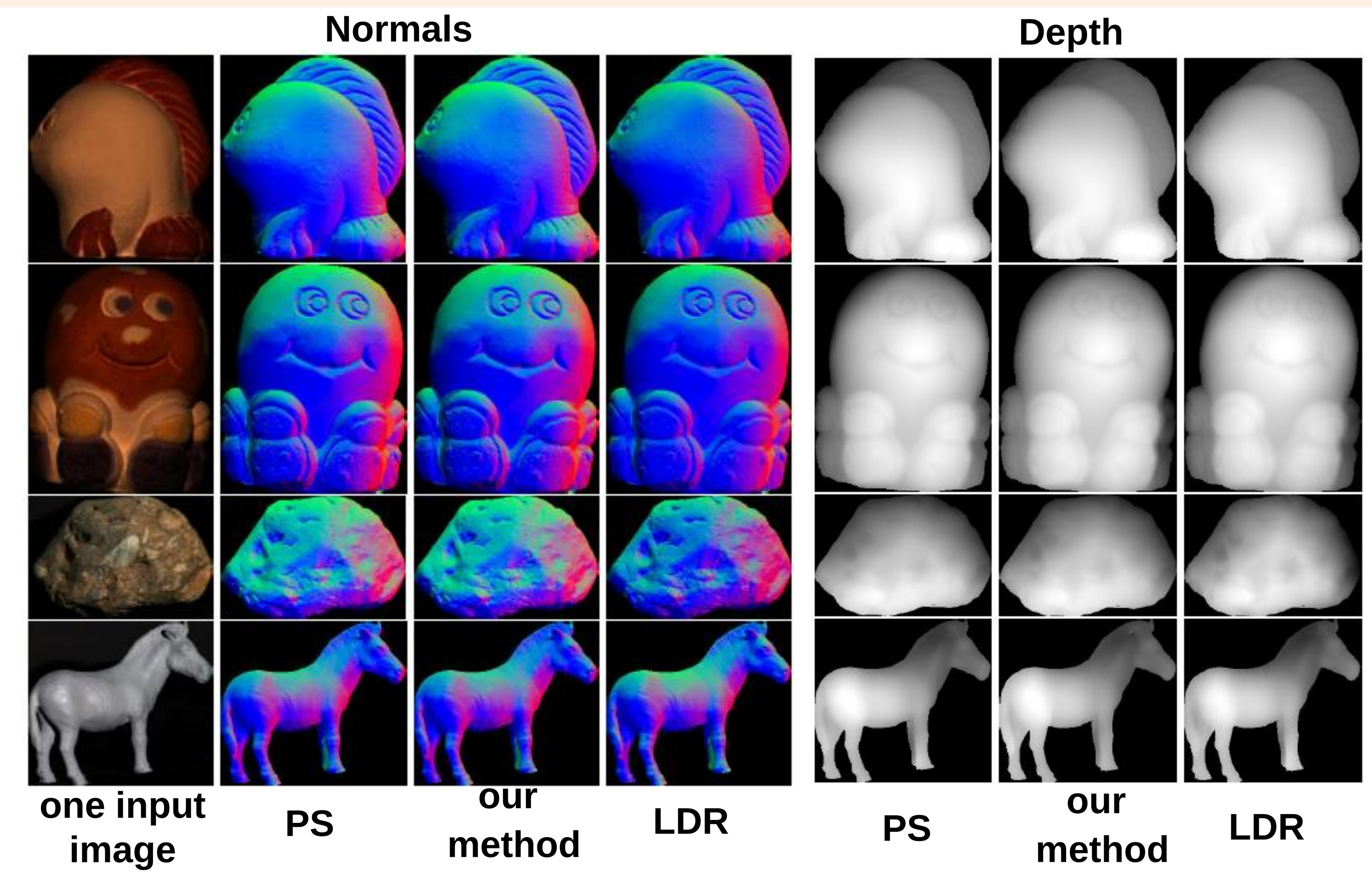
$$N \propto Q^T B \quad \text{and} \quad Q = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$$

$f \to \infty$: The last 3 columns vanish and yield 3 unknown parameters as in the classic GBR solution.

Results

Comparison with the state of art in orthographic UPS:

- **EM:** [N. Alldrin et al. Resolving the GBR ambiguity by entropy minimization, CVPR 2007]
- **SCPS:** [B. Shi et al. Self-Calibrating Photometric Stereo, CVPR 2010]
- **LDR:** [P.Favaro and T. Papadhimitri, A closed form solution to uncalibrated photometric stereo via diffuse maxima, CVPR 2012]



Dataset (Nr. Images)	A/E	Redfish (5)	Octopus (5)	Rock (12)	Horse (12)	Buddha (12)	Cat (12)	Owl (12)
EM Method	μ	8.63	9.03	22.16	20.65	15.05	15.39	18.48
	σ	1.14	0.76	1.88	3.85	2.19	3.78	5.58
SCPS Method	μ	7.60	13.23	24.88	21.01	13.58	6.15	10.47
	σ	4.32	9.85	7.42	9.57	4.93	2.83	4.75
LDR Method	μ	5.60	6.64	11.61	4.80	4.98	5.37	6.63
	σ	**0**	**0**	**0**	**0**	**0**	**0**	**0**
Our method	μ	**1.84**	**2.38**	**2.50**	**2.30**	**2.79**	**2.28**	**3.44**
	σ	**0**	**0**	**0**	**0**	**0**	**0**	**0**

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