Supplementary Material

A New Perspective on Uncalibrated Photometric Stereo

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1. Notes on Remark 1

In this section we show that our analysis is compatible with the perspective image irradiance equation of Tankus and Kiryati [1]:

$$\mathbf{I}_s(u,v) = \rho(u,v)\frac{(u-fp_s)\hat{p}+(v-fq_s)\hat{q}+1}{||L_s||\sqrt{(u\hat{p}+v\hat{q}+1)^2+f^2(\hat{p}^2+\hat{q}^2)}}. \tag{1}$$

In their work the relationship between real world and image coordinates is:

$$x = -\frac{u\cdot\hat{z}(x,y)}{f} \qquad y = -\frac{v\cdot\hat{z}(x,y)}{f}, \tag{2}$$

which is slightly different from ours:

$$x = \frac{u\cdot[\hat{z}(x,y)+f]}{f} \qquad y = \frac{v\cdot[\hat{z}(x,y)+f]}{f}. \tag{3}$$

One difference is the sign. They consider the projection center to be between the object and the image plane while we place it behind both the image plane and the object. A second difference is the reference frame. They consider a reference frame attached to the camera projection center while we attach it to the image plane. In our case the model more naturally converges to the orthographic projection when the focal length f tends to infinity. As a consequence of these changes, they define

$$\hat{p} \doteq \frac{z_u}{z} \qquad \hat{q} \doteq \frac{z_v}{z} \tag{4}$$

while in our analysis we have

$$\hat{p}_{our} \doteq \frac{z_u}{z+f} \qquad \hat{q}_{our} \doteq \frac{z_v}{z+f}. \tag{5}$$

To show that their model results in the same imaging equation that we derived in our submission, we rearrange the terms of their image irradiance equation as follows

$$\mathbf{I}_s(u,v) = \rho(u,v)\frac{(u-fp_s)\hat{p}+(v-fq_s)\hat{q}+1}{||L_s||\sqrt{(u\hat{p}+v\hat{q}+1)^2+f^2(\hat{p}^2+\hat{q}^2)}} = \tag{6}$$

$$= \rho(u,v)\frac{-fp_s\hat{p}-fq_s\hat{q}+(1+u\hat{p}+v\hat{q})}{||L_s||\sqrt{(u\hat{p}+v\hat{q}+1)^2+f^2(\hat{p}^2+\hat{q}^2)}} =$$

$$= \rho(u,v)\frac{-\frac{fp_s\hat{p}}{1+u\hat{p}+v\hat{q}}-\frac{fq_s\hat{q}}{1+u\hat{p}+v\hat{q}}+1}{||L_s||\sqrt{1+\frac{f^2(\hat{p}^2+\hat{q}^2)}{(1+u\hat{p}+v\hat{q})^2}}}.$$

Now, let us define

$$p \doteq \frac{-f\hat{p}}{1+u\hat{p}+v\hat{q}} \qquad q \doteq \frac{-f\hat{q}}{1+u\hat{p}+v\hat{q}}; \tag{7}$$

then, we have

$$\mathbf{I}_s(u,v) = \rho(u,v)\frac{pp_s+qq_s+1}{||L_s||\sqrt{p^2+q^2+1}} \tag{8}$$

which is the classical orthographic image irradiance equation in the new variables p and q. The analysis of the integrability constraint for this model follows quite closely the analysis that we have done on our model. The only differences are: 1) the sign of p and q are the opposite to the corresponding variables defined in our model; however, changing the sign to both p and q results in the same reconstructed normals except that the sign of the third coordinate is flipped. This flipping amounts to the concave-convex ambiguity that is already present in the original problem. 2) the range of the final reconstructed depth map is the same of the depth map we obtain in our model but shifted by the focal length f (see eq. 5).

References

[1] A. Tankus and N. Kiryati. Photometric stereo under perspective projection. *International Conference of Computer Vision*, pages I: 611–616, 2005. 1