Question # 1

Assume that x_0 is a solution to the following linear system

$$Ax_0 = b \tag{1}$$

where A is an $n \times m$ matrix, with $n < m$, x_0 is an m-dimensional column vector and b is an n-dimensional column vector.

1. Show that the L^2 norm of x_0 can be arbitrarily larger than the norms of A and b.

2. If $x^\perp$ is a vector orthogonal to x_0, is $x_0 + x^\perp$ also a solution to the above linear system?

Question # 2

Give an example of a linear system that has no solution.

Question # 3

Write the explicit formula of the gradient of

$$E[u] = \sum_{i=2}^{n-1} \sum_{j=2}^{m-1} \exp\left[-\frac{(u[i+1,j] - u[i,j-1])^2 + (u[i,j+1] - u[i-1,j])^2}{2\epsilon^2} \right] \tag{2}$$

with respect to the variable u, which is an $n \times m$ matrix, and where $\epsilon > 0$ is a given constant. Show all the steps of your calculations.

Question # 4

Write the explicit formula of the maximum likelihood estimator for the parameter $\alpha > 0$ of the following probability density distribution

$$p(x; \alpha, \epsilon) \propto \begin{cases} \alpha e^{-\alpha x} & x \geq \epsilon \\ 0 & x < \epsilon \end{cases} \tag{3}$$

given m independent and identically distributed samples $x^{(1)}, \ldots, x^{(m)}$. Show all the steps of your calculations. Do not just write the name of the formula.