Problem Set

This problem set is also a self-check for you to see your fit for this position. If you find these questions too "mathematical" or if you need to revise the books from your past courses or use internet resources, because these topics are no longer fresh in your mind, probably this phd position is not a good match for you.

Submit your answers typed (not handwritten). Use free tools for latex like overleaf.com if you do not have suitable software on your computer. Knowledge of latex is required for this doctoral position.

Question # 1

Suppose you only know the equation for computing the area of a square of side a, *i.e.*, Area(square) $= a \times a$. Suppose you also know how to calculate the sum of all the integers from 1 to n. Show that the area of an isosceles right triangle of side a is Area(triangle) $= \frac{a \times a}{2}$, by using the equation of the area of a square and the equation of the sum of n integers. Illustrate your procedure graphically.

Question # 2

Use the singular value decomposition to write explicitly all the solutions of a general underdetermined linear system.

Question # 3

Write the explicit formula of the gradient of

$$E[u] = \sum_{i=2}^{2} \sum_{j=2}^{2} \arctan(u[i,j] - u[i-1,j]) \tag{1}$$

with respect to the variable u, which is a 2×2 matrix. Show all the steps of your calculations.

Question # 4

You are given the following probability density distribution

$$p(x; \alpha, \epsilon) = \frac{1}{Z} e^{-\alpha x^2 - \epsilon}, \tag{2}$$

where Z is the partition function, and $\alpha > 0$ and ϵ are unknown scalars. Assume that you are given m independent and identically distributed samples $x^{(1)}, \ldots, x^{(m)}$. Write the explicit formula of the maximum likelihood estimator for the parameter α. Show all the steps of your calculations.